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The Contribution of Artificial Turf to Global Warming

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Abstract

This article discusses how the substitution of artificial grass for natural grass contributes to global warming. An algebraic model of the atmospheric transmittance in the infrared wavelengths from 0 to 15 microns is used to modulate the Planck law, yielding both the energy absorbed by the atmosphere and that transmitted through the atmosphere as a function of the ground temperature. The calculation shows that the energy absorbed by the atmosphere increases more rapidly than the amount transmitted through the atmosphere with increasing ground temperature. In situ experiments demonstrate that artificial grass reaches significantly greater temperatures than those reached by natural grass under the same meteorological conditions. As a result, artificial grass creates an additional amount of energy absorbed by the atmosphere. With the number of nationwide artificial grass installations, a typical result yields an additional energy deposited into the atmosphere during moderately warm summer days of 10 to 20 gigawatts. The annual nationwide cost savings to local governments by the substitution of artificial grass for natural grass is shown to be trivial.

Keywords: artificial grass; atmospheric transmittance; climate change; global warming; greenhouse effect; infrared window

Introduction

Background

In this paper we distinguish “artificial turf,” the term used in the industry, from “artificial grass.” The latter refers to the actual visible green plastic blades, which attain high temperatures under sunlit conditions, whereas the former includes infill and matting material. We prefer the term “artificial grass” and use it throughout as that which is not only visible but also relevant, as the radiating material, to the contribution to global warming.

Artificial grass was initially introduced for use in professional indoor athletic stadiums. Outdoor professional athletic stadiums, park dis-

tricts, and school districts followed this lead and adopted artificial grass fields for athletic purposes. As areas in the southwest of the United States suffer from drought conditions, a large market has developed for use of artificial grass for landscaping.

The manufacture of artificial grass requires plastics and heavy metals, which after a short lifetime are disposed, presenting environmental hazards. Decrying the use of plastics in general given their environmental dangers yet replacing natural grass with artificial, plastic, grass is inconsistent policy. Only by educating the public of the environmental consequences of installing artificial grass can this worrisome trend be mitigated or indeed terminated.

To document one aspect of this hazard, an algebraic model of the transmittance of radiation in the range of infrared wavelengths from 0 to $600\ \mu$ was created to determine the amount of energy flux both transmitted through the atmosphere and absorbed by the atmosphere as a function of the ground temperature. *In situ* studies have found a significant difference between the enhanced temperatures of artificial grass compared to natural grass under the same meteorological conditions. These data were used to help calculate the additional amount of energy radiated from the artificial grass and absorbed by the atmosphere compared to natural grass as a function of the ground temperature. A hypothetical case in which the

atmosphere absorbs a greater amount of radiation was also computed. Estimates show that the nationwide cost savings to local governments for the replacement of natural grass with artificial grass are relatively small. In addition, in contrast to the monochromatic transmittance utilized in the analysis, the total transmittance in the range of infrared wavelengths from 0 to 600 μ as a function of temperature for actual radiating blackbodies was determined to be about 26 percent.

Because artificial grass contributes to global warming in the United States, local, state, and federal policies need to be involved in efforts to reduce or ban their use. The rapid increase in the number of artificial turf fields being installed in Europe and the Asia-Pacific region warrants international attention. To mitigate the problem education about the negative effects and true costs of artificial grass will be important in changing behaviors of local officials on elected boards of local park districts and school districts as well as state and federal officials, all of whom could opt to retain natural grass. A numerical national rating system for artificial grass products based on life expectancy, chemical composition, and thermal behavior would help officials make more rational policy decisions regarding the use of artificial grass.

Environmental Problems with Artificial Grass Include Its Contribution to Global Warming

Various criticisms of using artificial grass include direct damage to the environment when natural grass and its inhabitants—insects as well as burrowing creatures such as worms—are killed; loss of the rainwater-absorbing quality of natural grass; loss of food source for birds;

and negation of the oxygen-producing function of natural grass (Kaminski, 2019; Peeples, 2017). To make room for the artificial grass, the existing soil is cleared from the installation site and discarded into a landfill (Guerriero, 2021). Rainwater does not penetrate as rapidly through artificial grass as it does through natural grass, resulting in decreased water entering watersheds and increased localized flooding (Peeples, 2017). Because natural materials such as grass absorb carbon dioxide during photosynthesis, replacing them with artificial grass also directly contributes to the increase of carbon dioxide (a primary greenhouse gas) in the atmosphere. The plastic and infill material of artificial grass does not provide either greenhouse gas capture or air-purifying services (Peeples, 2017).

Artificial grass can get hot, creating health problems for those walking or running on it (Guerriero, 2021; Peeples, 2017; G. Pulley, personal communication, January 28, 2020; Williams & Pulley, 2002). The elevated temperatures also increase the rate at which toxic gases such as benzothiozole and toluene are released from the artificial grass (Peeples, 2017).

Noting that not only does artificial grass have no climate benefit but also that production of the plastic blades emits carbon and uses fossil fuels, the UK Committee on Climate Change has recommended removing artificial grass fields and replanting natural grass, as well as planting trees, to help battle global warming (Kaminski, 2019). The common practice of replacing soil with sand to provide a more stable bed for the artificial grass also releases carbon dioxide stored in the Earth (Kaminski, 2019).

During the lifetime of artificial grass, the plastic blades are fractured, and those fragments become part of the environment. The toxic chemicals used as colorants (Action, 2013) create disposal problems for artificial grass when it has reached the end of its useful life. It is generally recognized that the elevated temperatures reached by artificial grass may contribute to global warming (Peeples, 2017). This article provides the first mathematical analysis of the severity of the problem. The analysis shows that a direct global warming results from radiation of additional energy flux into the atmosphere from the manufacture and installation of artificial grass. To those concerned about the health of the planet, this is worthy of discussion.

The replacement of natural grass by artificial grass leads to a decrease in the number of trees in urban forests, with severe environmental effects, including enhanced global warming. Trees, which are able to grow on natural grass, not only are natural refrigerators of their proximate environment by shading but also during photosynthesis absorb carbon dioxide, a significant greenhouse gas responsible for a major portion of the terrestrial greenhouse effect (Bordelon, n.d.). Trees purify the air by removing sulfur dioxide (SO_2), ozone (O_3), nitrous oxide (N_2O), and smoke particulates, in particular those generated by diesel engines (Bordelon, Urban Forestry Network). Along with water vapor, methane (CH_4), chlorofluorocarbon-12 (CCl_2F_2), and hydrofluorocarbon-23 (CHF_3), three of these substances filtered out of the air by trees, namely carbon dioxide, ozone, and nitrous oxide, are primary greenhouse gases (2007, NASA/Goddard Space Flight Center Conceptual Image Lab; Center for Climate and Energy

Solutions). They also provide food and habitat for insects, birds, and mammals. Trees do not grow in the plastic of artificial grass. Indeed, replacement of natural grass areas by artificial grass often entails destruction of trees, including entire stands of old growth trees.

This discussion considers how infrared radiation is transmitted through and absorbed by the atmosphere, which requires mathematical integration of the product of the Planck blackbody radiation law (see Appendix) with the transmittance of the atmosphere for radiating substances of varying temperatures. The radiating substances, in this case natural grass and artificial grass, are characterized by their thermophysical parameters. Analysis of these parameters, in particular the reflectance, specific heat capacity, and thermal conductivity, shows that artificial grass both absorbs more insolation energy from the sun and retains it to a greater extent than natural grass, leading to its elevated temperatures. *In situ* measurements by various groups confirm the extent of this excessive heating.

The model calculations (see Appendix) show that on a typical warm summer day, the energy flux absorbed by the atmosphere from artificial grass installations in the United States alone exceeds that absorbed from an equal area of natural grass by an amount equivalent to that of 10 to 20 moderately-sized nuclear power plants. This contribution to global warming continues to increase as an additional 1,200 to 1,500 sites replace natural grass with artificial grass annually in the United States (Lundstrom and Wolfe, 2019; Woodall, 2019). In addition, a frightening trend has appeared in which homeowners discard their natural

grass and install artificial grass to eliminate the need for lawn maintenance, conserve water, and to demonstrate to their family, neighbors, and friends that they are, ironically, “green.” This trend is particularly popular in areas of the southwest United States that are experiencing drought conditions.

The Thermal Properties of Artificial Grass

The thermophysics of artificial grass compared to natural grass indicates that the former will provide a relative source of heating. This is confirmed by independent *in situ* studies.

The relevant thermophysical parameters are the emissivity, the reflectance, the thermal conductivity, and the specific heat capacity. The emissivity of green natural grass and plastics are about the same. The former range from 0.95 for dry grass to 0.99 for green grass, with some dependence on season, whereas the emissivity for plastics generally is about 0.95. Natural grass is marginally able to radiate more efficiently. A major distinction resides with the reflective properties of artificial grass compared to that of natural grass. From about $0.7\ \mu$ to about $1.3\ \mu$, the reflectance of natural grass is about 0.60. Although it decreases at greater wavelengths, it remains greater than that of green artificial grass, which is about 0.06 out to $2.4\ \mu$ (Devitt *et al.*, 2007). The remainder of the insolation incident on artificial grass is absorbed.

Without an abundance of conducting free electrons, the thermal conductivity of both water and plastics is low. The thermal conductivity of water, the major constituent of natural grass, 70%, is 0.58 W/m-K, whereas the thermal conductivity of polyethylene and polypropylene, the olefin fibers out of which artificial grass is manufactured, ranges from

0.09 W/m-K to about 0.50 W/m-K, with many of their composites being in the 0.20 to 0.25 W/m-K range. The energy absorbed by the blades of artificial grass is relatively less efficiently conducted to its subsurface material. That subsurface material, similarly, made of plastics, is also a poor thermal conductor. In contrast, soil can be a relatively good conductor, with a thermal conductivity, depending on organic content, from 0.15 W/m-K to about 2 W/m-K, and, if saturated, from 0.6 W/m-K to about 4 W/m-K, for a total range of 0.15 to about 4 W/m-K.

Water has by far the highest specific heat capacity of any common substance, 4186 J/kg-K. Those of polyethylene and polypropylene range between 1700 and 1900 J/kg-K. This means that a large amount of thermal energy absorbed by natural grass can heat its water without the water temperature greatly increasing. Viewed alternatively, a given amount of thermal energy will cause a mass of polyethylene or polypropylene to increase its temperature by more than twice the temperature rise of an equal mass of water. In total, Devitt *et al.* (2007) found that more than 90% of the insolation heats the blades of the artificial grass with a resultant radiation into the atmosphere and less than 10% is conducted below the artificial grass material and into the soil.

Natural grass possesses another mechanism to remove heat, the evaporative cooling that results from transpiration and nighttime guttation, the expulsion of droplets of water. Neither mechanism is available to plastics.

Although the emissivity of natural grass is slightly greater than that of artificial grass, taken together consideration of the thermophysical

parameters indicates that under the same meteorological conditions artificial grass will attain higher temperatures than natural grass and largely retain those higher temperatures, in addition to the lack of transpiration and guttation. Because its temperature can rise significantly higher than natural grass, artificial grass radiates more energy flux into the atmosphere than natural grass. This leads to enhanced absorption of energy by the atmosphere in the range of wavelengths near the infrared window.

The actual contribution to heating of the atmosphere depends on the extent to which the temperature of artificial grass rises. The implications of these considerations of the thermophysics that greater temperatures are achieved by artificial grass compared to natural grass under the same meteorological conditions have been confirmed experimentally by several groups. They found comparable quantitative results.

Devitt et. al. (2007) measured the temperature of artificial grass in seasons of moderate temperatures. They found that the maximum surface temperature of the artificial grass

was approximately 38 K higher than that of natural grass and 34 K higher than the air temperature. The maximum artificial grass temperature recorded was 349 K (76° C), reached during the hottest summer months. Williams and Pulley (2002) performed similar measurements and had comparable results. They found that the grand mean temperature for artificial grass was 320 K (47° C), with a maximum mean in one hourly period of 343 K (70° C). The corresponding temperatures for natural grass were 22 K and 38 K lower, the latter figure agreeing with the finding of Devitt et. al. (2007). The highest temperature recorded for the surface of artificial grass was 367 K (94° C). McNitt and Petrunak (2016) measured a maximum surface temperature for artificial grass surfaces of 345 K (72° C), which was 41 K greater than the air temperature, while Buskirk et. al. (1971) found that the surface temperatures of artificial grass could exceed that of natural grass by temperatures from 35 K to 60 K, recording a maximum temperature for artificial grass of 333 K (60° C).

From these studies, one can reasonably and conservatively suggest that

artificial grass can attain temperatures 30 K above that of natural grass in the same environment. Particular differences depend on the artificial grass product, namely, its material and construction, location, season, and time of day. These maximum surface temperatures suggested model calculations up to 370 K. Thermophysics and the lack of water to provide evaporative cooling can explain the phenomenon. In short, after being heated by sunlight, the temperature of artificial grass rises more quickly than that of natural grass, and that elevated temperature is maintained.

Results and Discussion

Numerical and Graphical Results

Details of the calculation of energy flux absorbed in the atmosphere from artificial grass compared to natural grass are provided in the Appendix. We report the results here.

Due to partial opacity in the infrared wavelengths, a portion of the energy radiated from surfaces of given temperatures is absorbed in the atmosphere. Table 1 shows the radiation

Table 1. The Energy Flux Emitted by Blackbodies at Temperatures from 275 K to 370 K^a

T (K)	F _{BB} (watts/m ²)	F _{tr} (watts/m ²)	F _{abs} (watts/m ²)	T (K)	F _{BB} (watts/m ²)	F _{tr} (watts/m ²)	F _{abs} (watts/m ²)
275	323.8	75.1	248.8	325	631.7	170.7	461.1
280	348.0	82.4	265.6	330	671.5	183.2	488.3
285	373.6	90.2	283.4	335	713.1	196.3	516.8
290	400.5	98.5	302.0	340	756.7	210.1	546.6
295	428.8	107.2	321.6	345	802.2	224.4	577.8
300	458.6	116.5	342.2	350	849.7	239.4	610.3
305	490.0	126.2	363.8	355	899.3	255.1	644.2
310	522.9	136.5	386.4	360	951.0	271.5	679.6
315	557.5	147.3	410.1	365	1005.0	288.5	716.5
320	593.7	158.7	435.0	370	1061.2	306.3	754.9

^aBased on the findings of Buskirk et. al. (1971), Devitt et. al. (2007), McNitt and Petrunak (2016), and Williams and Pulley (2002) for the highest temperatures attained by artificial grass, the calculations extend to 370 K.

emitted by blackbodies in the wavelength interval 0 to $600\ \mu$ at temperatures from 275 K to 370 K. This radiation is only partially transmitted through the atmosphere. The differences between the energy flux absorbed and transmitted, as given in Table 1, are presented in Table 2. The curves shown in Figure 1 were created by drawing smooth lines through the results of the calculations at 5 K intervals of equation (5), equation (6), equation (7), and equation (8) (see Appendix), as provided by the data in Table 1 and Table 2. In Table 2, the second and fifth columns provide the results for the current abundance of greenhouse gases in the atmosphere. The third and sixth columns present the results if the transmission through the atmosphere were decreased by 50 percent, to be discussed below.

The behavior of the absorbed and transmitted radiation results from two effects. First, as the temperature increases, the amount of blackbody radiation increases. The behavior,

however, results from the nature of the transmittance function (see Figure 5 and Figure 6). Figure 4 shows that the absorbed radiation increases with temperature more rapidly than does the transmitted radiation, that which escapes through the atmosphere. These differences between the amounts of radiation absorbed and transmitted as a function of temperature, are indicated by drawing smooth lines through the results presented in Table 2.

The Magnitude of the Effect

Consider a day in which the temperature reaches only a moderately warm 81°F , that is, 300 K. From Figure 2 (see Appendix), if the artificial grass reaches a temperature 35 K higher than the natural grass, then the amount of energy flux absorbed by the atmosphere is about 175 watts/m^2 greater than would be absorbed from the radiation from natural grass. At the temperature of 300 K, then, a playing field measuring 100×100 meters deposits into the atmosphere of about 1.75 megawatts

more (calculated as: $175\text{ watts/m}^2 \times 10^4\text{ m}^2 = 1.75$) by absorption in the wavelength interval 0 to $600\ \mu$ than the same size natural grass field. This is the order of magnitude of the power-generating capacity of a solar photovoltaic power plant. Based on the estimated 13,000 artificial grass surfaces in the United States, these artificial grass surfaces deposit 2.3×10^{10} watts more energy flux than that generated by natural grass, equivalent to 23 moderately-sized nuclear power plants. Although this figure is small compared with the total energy budget of the atmosphere, it remains significant as an additional source of global warming. (Chestney & Januta, 2021).

Because of difference in sizes of installed artificial grass playing fields, variation in materials used by manufacturers, and climate variations seasonally, daily, and across the country, these figures should be considered only as an order of magnitude estimate. To compensate for cloud cover, a conservative estimate of nationwide additional power output from artificial grass is about the equivalent of ten such power plants.

The problem does not result from the magnitude of the energy deposited by absorption of the radiation emitted by artificial grass, but rather that these emissions are another source of global warming. Adding such another source is folly. Although governments try to fight global warming by reducing the magnitude of its various sources, every additional amount of energy deposited into the atmosphere must be balanced by some process to remove energy. None, though, exist.

We note that even at the current rate of installation of artificial grass playing fields, the acreage in the foreseeable future is much less than the

Table 2. The Difference in the Energy Flux Absorbed by the Atmosphere and Transmitted through the Atmosphere as a Function of the Temperature of the Radiating Blackbody

T (K)	ΔF Current (watts/m ²)	ΔF Projected (watts/m ²)	T (K)	ΔF Current (watts/m ²)	ΔF Projected (watts/m ²)
275	173.7	248.8	325	290.4	461.1
280	183.2	265.6	330	305.1	488.3
285	193.2	283.4	335	320.5	516.8
290	203.6	302.0	340	336.6	546.6
295	214.4	321.6	345	353.3	577.8
300	225.7	342.2	350	370.8	610.3
305	237.5	363.8	355	389.1	644.2
310	249.9	386.4	360	408.1	679.6
315	262.8	410.1	365	428.0	716.5
320	276.3	435.0	370	448.7	754.9

Note: These results are presented graphically in Figure 3. The results in columns 2 and 5 are derived from the data presented in Table 1 and are presented in graphical form in Figure 1.

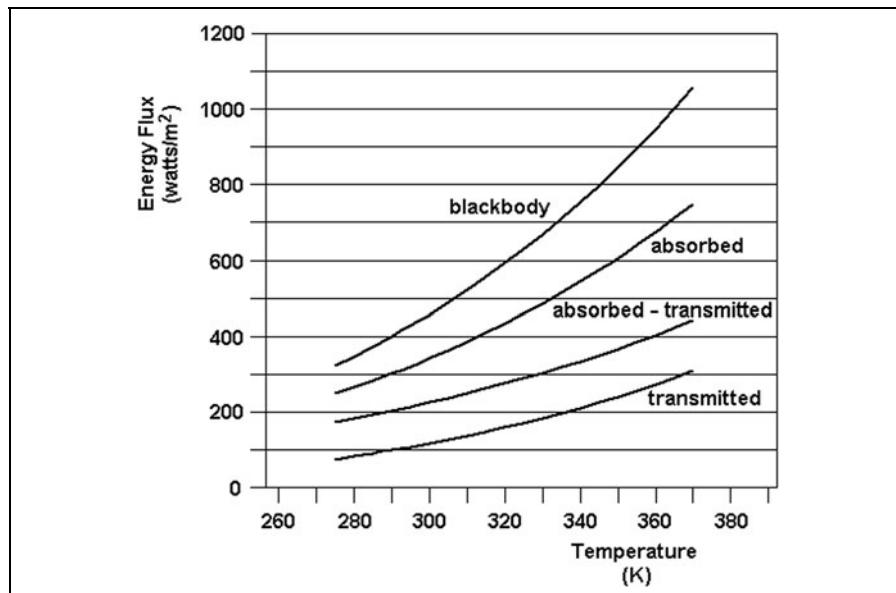


Figure 1. Energy radiated from surfaces of given temperatures based on approximately the abundance of greenhouse gases that exist today. The results of the calculation depicted in Figure 6 (see Appendix) falls on these curves at 300 K, with the result for the energy absorbed at 300 K as noted in Table 1 being 342.2 watts/m².

acreage of other heat-generating surfaces in urban heat islands such as concrete, asphalt, and roofing material. Although *in situ* studies show that concrete and asphalt in fact at-

tain temperatures significantly lower than that attained by artificial grass under the same daytime temperatures (Devitt *et. al.*), because of such differences in areal coverage the

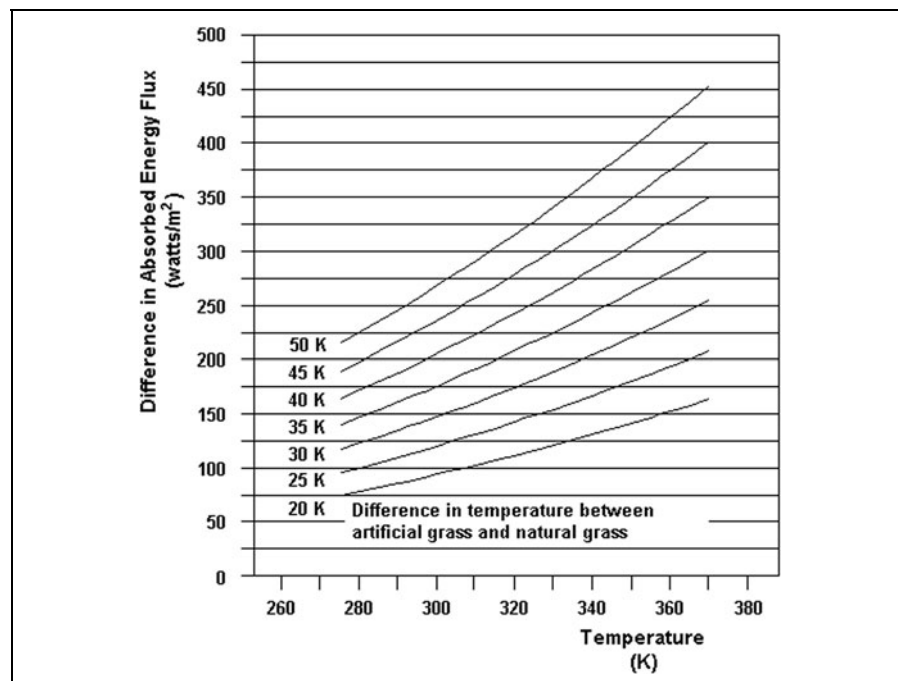


Figure 2. The difference in absorbed energy flux created by artificial grass and natural grass. The difference in energy flux is shown for values of ΔT from 20 K to 50 K as a function of the temperature of the natural grass. This result leads to the conclusion that artificial grass contributes to global warming.

contribution of the latter surfaces to direct heating of the atmosphere thereby far exceeds that of artificial grass playing fields. In addition, as noted, the direct contribution of urban heat islands to global warming is much less than the indirect cause of greenhouse gases. The concern lies in the installation of environmentally-harmful artificial grass being optional.

Conjectures: Feedback Loops and the Future Atmosphere of the Earth

The calculations presented in the foregoing were performed using the transmittance as defined in Table 3, the current transparency in the infrared region near the infrared window of the atmosphere. As additional greenhouse gases are deposited into the atmosphere, global warming results from a decrease in this transparency of the atmosphere and an associated increase in absorption.

As evidenced in Figure 5 (see Appendix), the effects are most prominent with water vapor and carbon dioxide. As the atmosphere warms, it creates a feedback loop, leading to additional deposits of water vapor. As the temperature rises, more water vapor enters the atmosphere through evaporation. The warmer atmosphere can retain the H₂O in the water vapor phase. The H₂O lines get deeper because of its increased abundance, and wider, because of increased thermal broadening with the increased temperature, further enhancing the ability of the H₂O lines to absorb. The equivalent width of the lines thereby increases. As more CO₂ enters the warmer atmosphere because of human-made emissions its absorption lines also get deeper and broader. These deeper and wider absorption lines of H₂O and CO₂

Table 3. The Algebraic Expressions Fit to the Transmittance Function Model (see Figure 5, bottom graph)

Wavelength Range (microns)	Factor
$\lambda \geq 0.1$ and $\lambda \leq 1.2$	0.62
$\lambda \geq 1.5$ and $\lambda \leq 1.9$	0.76
$\lambda \geq 2.0$ and $\lambda \leq 2.5$	0.79
$\lambda = 3.0$ or $\lambda = 3.1$	0.28
$\lambda > 3.1$ and $\lambda < 3.5$	0.42
$\lambda \geq 3.5$ and $\lambda \leq 4.1$	0.90
$\lambda \geq 4.5$ and $\lambda \leq 5.4$	$(385.0 - 70.0 \lambda)/100$
$\lambda \geq 7.7$ and $\lambda < 8.4$	$(-745.0 + 98.0 \lambda)/100$
$\lambda \geq 8.4$ and $\lambda < 10.8$	0.78
$\lambda \geq 10.8$ and $\lambda < 13.4$	$(224.0 - 13.5 \lambda)/100$
$\lambda \geq 13.4$ and $\lambda \leq 13.9$	$(787.5 - 56.3 \lambda)/100$

Note: The transmittance equals 0, indicating 100 percent opacity, except as noted. The factors define the atmospheric transmittance function in the infrared, $T(\lambda)$.

mean more radiation is being absorbed by the atmosphere and the atmosphere increases in warmth. A feedback occurs.

Changes in the atmosphere can be codified by decreasing the values defining the *current* transmittance shown in Table 3. This will result in changes in Figure 1 and Figure 2. The energy flux transmitted through the atmosphere from 0 to 600μ will decrease and the energy flux absorbed will increase, leading to a net increase in the energy flux absorbed.

To show the effect of increasing abundances of water and carbon dioxide in the atmosphere, modifications to the model were made. The transparency was decreased arbitrarily by 50 percent in each of the wavelength segments of the transmission function defined in Table 3. This change decreases the amount of radiation transmitted through the atmosphere by 50 percent, and because the radiation emitted by the blackbody remains the same, increases the amount of radiation ab-

sorbed by the atmosphere by the same numerical amount by which the radiation transmitted was decreased. The results are shown in Figure 3.

Figure 4 compares the difference between the energy flux absorbed by the atmosphere and that transmitted through the atmosphere as a function of temperature for the two cases. The results on which these curves are based are calculated from equation (8) (see Appendix) and presented in Table 2. Although the results shown Figure 3 are hypothetical, they are reminders of what we might be doing to the Earth and the need to prevent increased global warming in both its direct and indirect modes.

Comparison of Strategies to Reduce the Effects on Global Warming

The total surface area of fields converted to artificial grass is orders of magnitude less than that of reflective surfaces such as asphalt roads, automobile roofs, building rooftops, and the like. These, however, are

relatively permanent features of our civilization, whereas the trend to convert natural grass surfaces to artificial grass can be mitigated and can have an immediate effect. Asphalt roads and parking lots in urban heat islands will be with us as long as we have automobiles. Advances in engineering design of asphalt and use of reflective colors to decrease the absorption of sunlight by roads and buildings promise to reduce the heat contribution from new construction in urban heat islands. Actual conversion of a significant portion of existing infrastructure, however, is financially untenable for municipal governments and private developers.

The indirect contribution of greenhouse gases won't, even under optimistic models, be mitigated for decades if not centuries or millennia (Chestney & Januta, 2021), if ever. Knowledge of the effect of replacing artificial grass with natural grass contrast, can lead to a reduction in such actions within only a few years, including replanting of natural grass when the first generation of artificial grass fields must be replaced. The contribution of artificial grass can be reduced or eliminated in the time cycle of the expected life of artificial grass, within a generation. Although the contribution of artificial grass is small compared to the contribution of urban heat islands, and the contribution of urban heat islands is small compared to that of greenhouse gas emissions, every contribution to global warming is significant (Chestney & Januta, 2021).

Replacing artificial grass with natural grass would not require diverting significant funds from those earmarked for reducing the major contributions to global warming, the anthropomorphic creation of greenhouse gases. The

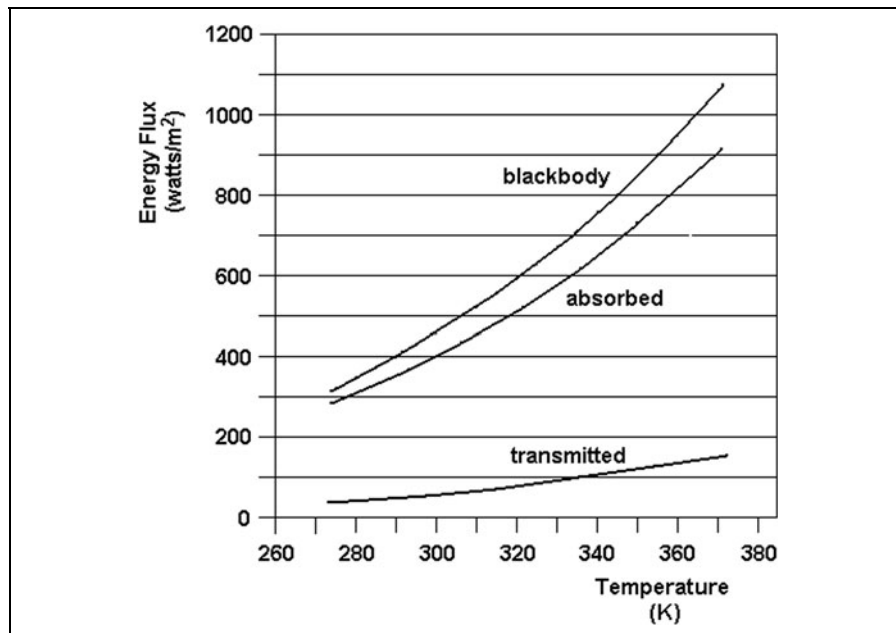


Figure 3. The portion of the energy radiated from surfaces at given temperatures that would be absorbed if the transmission of energy flux through the atmosphere were decreased by 50 percent. To avoid clutter on the graph, the difference between the energy flux absorbed and that transmitted is not displayed.

major cost would be disposal of the existing artificial grass fields. The cost of removal and disposal into a landfill is approximately \$65 per ton (Woodall, 2019). The material weighs about 0.5 lb/ft², or 5.4 lb/m². The total weight of a 100-square meter playing field then would be about 54,000 lb, with a disposal price tag of about \$1,800. This cost could be borne by local school districts and park districts; it would not funnel money away from the federal government, energy supplier research programs working to reduce greenhouse gas emissions, or automobile manufacturers.

Decrying the use of plastics in general for their environmental dangers yet replacing natural grass with artificial, plastic grass is inconsistent policy. Only by educating the public of the environmental consequences of artificial grass and changing government policy can this worrisome trend be mitigated or indeed terminated.

The Trivial Cost Savings with Artificial Grass

The rationale for government officials to rip up natural grass and its insect inhabitants (Kaminski, 2019) and install artificial grass resides in reducing maintenance costs and the cost of watering, as well as the need to conserve water in the increasingly drought-stricken western and southwestern states. Outdoor sports stadiums may be open year-round in the southern and western states but only six months long in the northern and eastern states. Thus, nine months can be used as a representative average to estimate cost savings. An estimate of maintenance for natural grass includes the cost of water at \$10 per thousand gallons and a weekly watering that uses 1,000 gallons. This estimate does not factor in rain, the cost of commercial mowing (which averages \$50 per each mowing of natural grass), or the once-per-year aeration and fertilization costs. Using these generous

numbers, maintenance is about \$2,200 annually per field. Based on the total of about 13,000 artificial grass playing surfaces in the United States, the annual nationwide cost savings based on maintenance and water usage can be estimated as:

Annual Savings

$$\begin{aligned}
 &= 1.3 \times 10^4 \text{ fields} \\
 &\quad \times \$60 / (\text{week-field}) \times 36 \text{ weeks} \\
 &= \$28,000,000,
 \end{aligned}$$

which is likely an overestimate. This cost is trivial compared to the potential economic, social, and demographic costs of global warming, to which the replacement of natural grass by artificial grass contributes. This realization by local governments could deter them from such expensive action (Golden, 2013). Although a more robust analysis would follow, after Loss et al. (2014), the multiplicative Monte Carlo analysis presented by Golden (2021), in which distributions of both the sizes of artificial grass fields and their thermal behavior could be analyzed to produce a more precise value for the energy flux deposited by artificial grass fields above that generated by natural grass fields nationwide, the relatively small contribution to global warming does not justify such an additional analysis.

McNitt and Petrunak (2016) in their discussion of the higher temperatures reached by artificial grass note that “some organizations have installed irrigation systems to reduce the heat” and that “irrigation of these fields dramatically reduce (sic) the surface temperature,” but their testing showed that “a dramatic reduction in temperature is short term.” Williams and Pulley (2002) found the same short-term effect for irrigation on reducing the temperature. Some manufacturers of artificial grass actually recommend spraying water on artificial grass to wash off dog feces

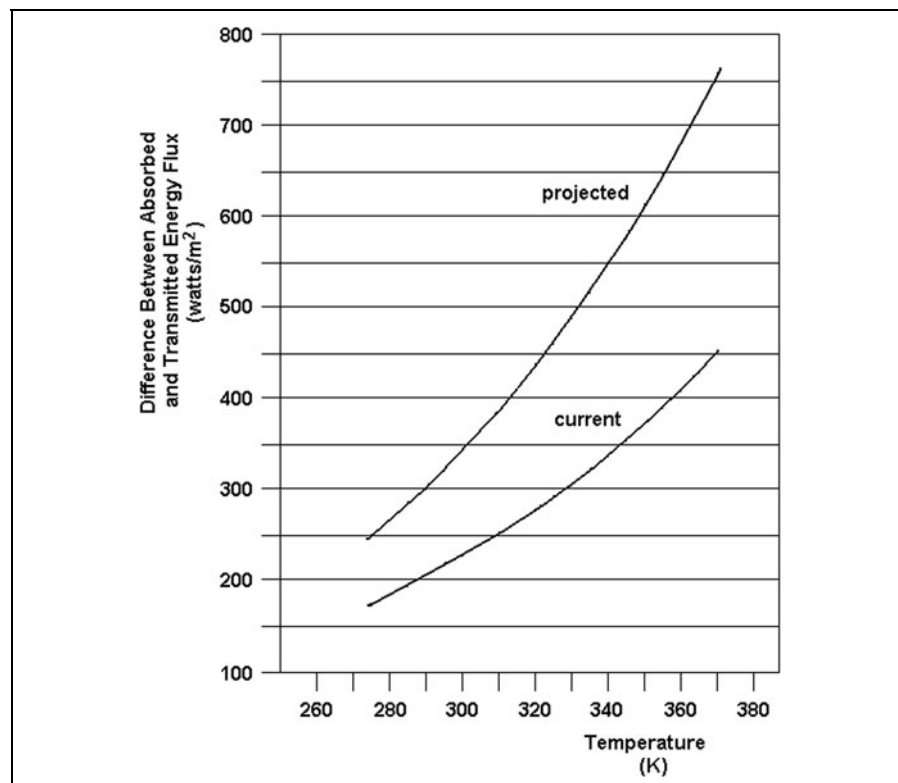


Figure 4. The difference between the energy flux absorbed in the atmosphere and transmitted through the atmosphere in the range of infrared wavelengths from 0 to $600\ \mu$ increases markedly with increasing temperature.

The results based on the infrared window as presented in Figure 5 and Figure 6 (see Appendix) are referred to as "current." They are presented as the "absorbed-transmitted" curve in Figure 1. Those referred to as "projected" are results based on a hypothetical case in which the wavelength region near the infrared window becomes more opaque than is currently the case, thereby decreasing the amount of energy flux that is transmitted through the atmosphere.

and other contaminants and to reduce the elevated temperatures that cause artificial grass to release toxic chemicals, such as benzothiazole and toluene, that are released from some artificial fields (Peeples, 2017). Ironically, the cost and water-saving benefits claimed by these organizations are attributable largely from not sprinkling water on natural grass.

Final Thoughts

Those concerned with climate change routinely ignore the result of replacing natural grass with artificial grass as part of the urban heat island. Although this is understandable, its contribution being much smaller than the contribution of greenhouse

gases created by fossil fuels, that plastic and infill material must also be considered in the context of its life cycle and, in the more general sense, the environmental impact of all products that we produce, use, and discard. This article highlights the need for critical involvement in our choices for what we use and consume, from the individual homeowner to levels of government. That involvement must include not only the immediate effects of our actions but also the effects of the life cycle of a product, from its manufacture to its disposal. Only then can we attain a state of sustainability.

As an example of this need for critical involvement, we recognize the attrac-

tiveness of replacing natural grass by artificial grass in drought-stricken areas, such as the American southwest, and the consequent saving of water used for irrigation of the natural grass. For such landscaping, use of the traditional rock gardens, natural low-water-use flora such as cactuses and succulents, and drought-resistant grasses, the most drought-resistant being buffalo grass, can greatly reduce the water consumption and should be considered. Indeed, such flora have adapted to high temperature climates and can be expected to survive under all but unimaginably extreme global warming.

We have only one Earth. It is our responsibility, for ourselves, our future generations, and indeed for all the flora and fauna that inhabit the Earth that we attain that state of sustainability.

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Appendix

Absorption of Terrestrial Blackbody Radiation by the Atmosphere

The intensity of energy radiated per second from a unit area of an idealized object at temperature T in a wavelength interval $\Delta\lambda$ centered at λ into a unit solid angle is given by Planck's blackbody radiation law,

$$B_{\lambda}(T) = \left[\frac{2hc^2}{\lambda^5} \right] \frac{1}{e^{hc/k\lambda T} - 1}, \quad (1)$$

where h is Planck's constant, $h = 6.63 \times 10^{-34}$ joules-s, c is the speed of light in a vacuum, $c = 3.0 \times 10^8$ m/s, and k is Boltzmann's constant, $k = 1.38 \times 10^{-23}$ joules/K. The Stefan-Boltzmann law provides the energy radiated per second per unit area over all wavelengths, referred to as the radiative energy flux, $F(T)$, or simply the energy flux,

$$F(T) = \sigma T^4 \quad (2)$$

where the Stefan-Boltzmann constant $\sigma = 5.67037 \times 10^{-8}$ watts/m²-K⁴ and T is the temperature in

Kelvin. Our quoting σ to six significant figures is explained below. Wien's displacement law provides the wavelength at which the amount of radiation is maximum for a blackbody of temperature T ,

$$\lambda_{\max} = \frac{0.00290}{T} \text{ m-K}, \quad (3)$$

where T is in K.

Planck's law, equation (1), provides the intensity of radiation which is radiated per unit solid angle. The radiation, assumed isotropic, which is emitted into 2π solid angles vertically, in the outward direction, and is therefore relevant here is the monochromatic radiative (or radiant) energy flux, or simply the radiative (or radiant) energy flux, viz.

$$B'_{\lambda}(T) = \pi B_{\lambda}(T) \quad (4)$$

Henceforth, we will forego the additional notation of $B'_{\lambda}(\lambda)$, and will use $B_{\lambda}(T)$ to refer to the observable, the monochromatic radiative energy flux or, more simply, the radiative nature being understood, the monochromatic energy flux.

Modeling the Region near the Infrared Window

The top graph in Figure 5 shows the transmittance in the atmosphere of infrared wavelengths 0 through 15 μ . The atmosphere is largely opaque to infrared radiation, resulting in the heat radiation being absorbed. A small region of the infrared portion of the spectrum, between 7.7 and 14 μ wavelength, is partially transparent, the so-called infrared window. From about 14 to beyond 1,000 μ , or 1 mm, the atmosphere is essentially 100 percent opaque. In the bottom graph in Figure 5 the graphical representation of an algebraic model, $T(\lambda)$, is superimposed for the transmittance function, the details of which are provided in Table 3.

Radiation Transmission and Absorption

The energy flux radiated outward in a wavelength interval of interest, λ_1 to λ_2 , by a blackbody at temperature T is obtained by a simple integration:

$$F_{BB}(T) = \int_{\lambda_1}^{\lambda_2} B_{\lambda}(T) d\lambda, \quad (5)$$

(Appendix continues $\rightarrow$)

where $B_\lambda(T)$ is the monochromatic radiative energy flux as defined by equation (4). To determine the amount of the energy radiated in the 0 to 600 μ region that is transmitted through the atmosphere, the product of the transmittance function and the Planck law are numerically integrated, as follows:

$$F_{tr}(T) = \int_{\lambda_1}^{\lambda_2} T(\lambda) B_\lambda(T) d\lambda \quad (6)$$

That radiation not transmitted is absorbed by the atmosphere. The amount of radiation over these wavelengths that is absorbed, $F_{abs}(T)$, is then, using equation (6),

$$\begin{aligned} F_{abs}(T) &= F_{BB}(T) - F_{tr}(T) \\ &= \int_{\lambda_1}^{\lambda_2} [1 - T(\lambda)] B_\lambda(T) d\lambda \end{aligned} \quad (7)$$

The difference in the amount absorbed and transmitted in this infrared wavelength region is:

$$\Delta F(T) = F_{abs}(T) - F_{tr}(T) \quad (8)$$

and the ratio of the energy flux transmitted through the atmosphere to the energy radiated by the blackbody is:

$$R(T) = \frac{F_{tr}(T)}{F_{BB}(T)} \quad (9)$$

Radiation emitted in directions other than the vertical will pass through greater path lengths before leaving the atmosphere and will therefore be more greatly absorbed. All absorbed radiation, not simply that traveling in a vertical, outward, direction, will heat the atmosphere. As a result, the numerical results presented here are underestimates of the amount of radiation that is absorbed.

We integrate from 0 to 600 μ , in the far infrared, including all but the long wavelength tail of the blackbody

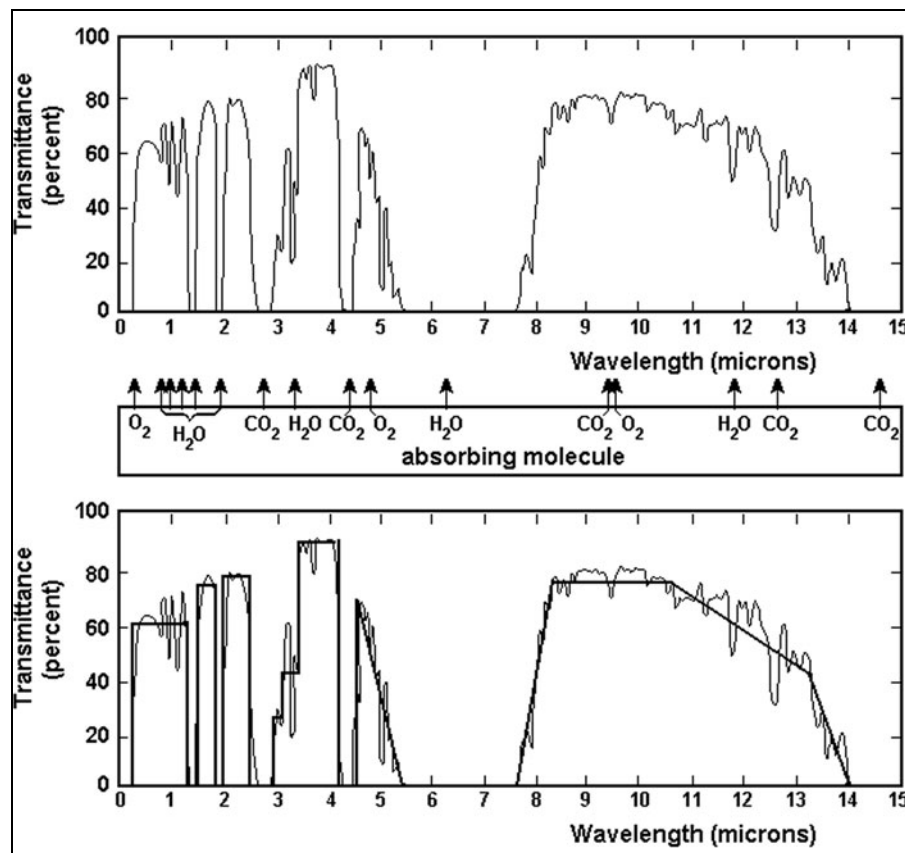


Figure 5. The graph on the top shows the percent of radiation transmitted through the atmosphere. The graph on the bottom shows the numerical model, $T(\lambda)$, the transmittance function based on calculations by the author. Adapted from U. S. Naval Academy, usna.edu.

curve. An interval of $\Delta\lambda = 10^{-7}$ m, corresponding to 0.1 μ , is used in the numerical procedure, with the transmittance function $T(\lambda)$ evaluated at the center of the intervals. The first interval, for example, from 0 to 0.1 μ , corresponding to 1×10^{-7} m, is in this way evaluated at 0.05 μ . The calculation is performed at 5 K intervals for temperatures from 275 K to 370 K, corresponding to 2 °C to 97 °C, which includes the range of temperatures that artificial grass surfaces will attain on a sunlit summer day. This can be up to 40 K greater than the ambient temperature.

To estimate the error in the numerical integration, the result for the total radiation emitted by the black-

body, σT^4 , equation (2), is compared to the result of the integration for a blackbody at the same temperature. Using the value of σ to six significant figures, errors of only 0.14 percent were found to be a result of this numerical technique.

Figure 6 shows the situation described in equation (6) in graphical form for a blackbody at the temperature of 300 K, about 81° F, a typical temperature for a warm summer day. Although much of the transmission of radiation occurs in the infrared window between wavelengths of 7.7 and 14 μ , the peak for a blackbody at higher temperatures will migrate as given by equation (3) (see Appendix) to smaller wavelengths, placing it in

(Appendix continues →)

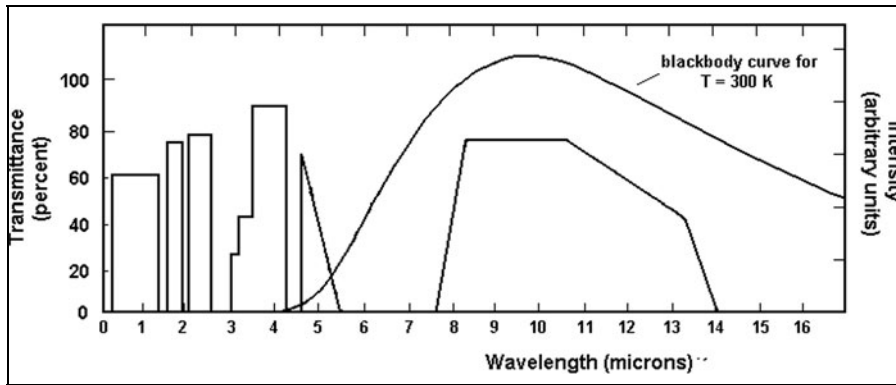


Figure 6. To determine the amount of radiation that is absorbed by the atmosphere in the range of infrared wavelengths of interest, first the amount that is transmitted is determined by multiplying the Planck blackbody curve at given temperatures by the transmittance function, as modelled piecewise, and integrate from 0 to 600 μ , equation (6). Then equation (7) gives the amount absorbed. The graph shows the scenario for a blackbody at a temperature of 300 K, about 81 °F. Here the peak of the blackbody curve occurs at 9.7 μ wavelength, just long of the wide wavelength interval of zero transmittance. Much of the energy emitted by such a blackbody is absorbed by the atmosphere.

the middle of the 5.5 to 7.5 μ interval of zero transmittance. As a result, increasingly larger amounts of energy will be absorbed from objects radiating at increasingly higher temperatures.

Determination of the best fit model

To determine the equation which best fits the results for the absorption of energy as a function of temperature in the range of wavelengths from 0 to 600 μ , provided in the fourth and eighth columns of Table 2 it is reasonable based on physical considerations to assume a form

$$F_{abs}(T) = a_1 e^{a_2 T} \sigma T^4, \quad (10)$$

where σ is the Stefan-Boltzmann constant and a_1 has the units of watts/ m^2 . Using the results presented in the fourth and eighth columns of Table 2, shown as the curve labeled “absorbed” in Figure 1, we find the best fit to the absorption results is given by

$$\begin{aligned} a_1 &= 0.891 \\ a_2 &= -6.424 \times 10^{-4} K^{-1} \end{aligned}$$

with a standard error of estimate, defined below, of 2.1 watts/ m^2 based on two degrees of freedom resulting

from the use of the two parameters. This small standard error of estimate, compared to the values of absorption to which the fit is imposed, indicates that equation (10) provides an excellent fit.

Similarly, a fit of the same form as equation (10), viz.

$$F_{tr}(T) = b_1 e^{b_2 T} \sigma T^4, \quad (11)$$

can be made to the transmitted results, presented in the second and fifth columns of Table 2, shown as the curve designated as “transmitted” in Figure 1. We find the best fit to these transmission results is given by

$$\begin{aligned} b_1 &= 0.161 \\ b_2 &= 1.652 \times 10^{-3} K^{-1} \end{aligned}$$

with a standard error of estimate of 2.4 watts/ m^2 based on two degrees of freedom resulting from the use of the two parameters. As with the absorption data, this small standard error of estimate, compared to the values of transmission to which the fit is imposed, indicates that the form of equation (10) and equation (11) provides an excellent fit.

The difference, equation (8), a measure of the energy being deposited into the atmosphere compared to that being transmitted, over the range of temperatures 275 K to 370 K, is given then by

$$\Delta F(T) = (a_1 e^{a_2 T} - b_1 e^{b_2 T}) \sigma T^4$$

from equation (10) and equation (11).

By equation (10), we can then calculate the difference in the energy flux absorbed by the atmosphere in the range of wavelengths from 0 to 600 microns between that radiated by artificial grass and that radiated by natural grass for various differences in temperatures. If ΔT is the (positive) difference between the temperature reached by artificial grass compared to that reached by natural grass under the same meteorological conditions, then we can use equation (10) to calculate the increase in the energy flux absorbed by the atmosphere in the infrared from 0 to 600 μ as

$$\begin{aligned} \Delta F_{abs}(T) &= a_1 e^{a_2(T+\Delta T)} \sigma (T+\Delta T)^4 \\ &\quad - a_1 e^{a_2 T} \sigma T^4, \end{aligned} \quad (12)$$

where T is the temperature of the natural grass and σ is the Stefan-Boltzmann constant. Figure 5 shows the results for a range of six values of ΔT in 5 K increments from 20 K to 50 K, which bracket the values found by Devitt *et al.* (2007), Williams and Pulley (2001), and Buskirk *et al.* (1971). These results lead to the conclusion that artificial grass contributes to global warming.

The natural transmittance of the atmosphere

With these results, we can revisit the transmittance of the atmosphere in the infrared. Figure 5 and Figure 6

(Appendix continues $\rightarrow$)

presented the transmittance in the wavelengths near the infrared window as usually presented, as *monochromatic* transmittance. This, however, is relevant for spectral observations of celestial objects in infrared astronomy. Real, natural objects radiate approximately as blackbodies, and that is our concern here. In that context, the meaningful transmittance should be discussed relative to the temperature of the blackbody, not a particular frequency or narrow bandpass. Figure 4 presents the ratio, $R(T)$, of $F_{tr}(T)$, the energy flux generated by blackbodies which is transmitted through the atmospheric infrared window, to $F_{BB}(T)$, the energy flux of the blackbody, as a function of temperature of the blackbody, equation (9). We can refer to this ratio as the “natural transmittance” of the atmosphere. The values of the ratios $R(T)$, deduced from the results presented in Table 2, are, as seen, small numbers.

Although the interval between 7.7 and $14\ \mu$ may be referred to as the infrared “window,” we see that, for natural radiating objects, the wavelength region between 0 and $14\ \mu$ cannot be described as transparent. Despite the ordinate scale in Figure 5

being expanded to show detail, one can state that for objects radiating at terrestrial temperatures the natural transmission through the portion of the infrared from 0 to $600\ \mu$ is $26 \pm 3\%$.

This contrasts greatly with the monochromatic transmittance detailed in Figure 5 and Figure 6, as represented algebraically in Table 3. We see that the window has regions of transmittance of up to 90%, between 3.5 and $4.1\ \mu$, with other wavelength regions having a transmittance above 75%. Yet, for real, natural radiating objects, the transmittance, as a function of the blackbody temperature, is about 26%. We see that, in fact, for natural objects, in the wavelength range from 0 to $600\ \mu$, the atmosphere absorbs about 74% of the energy flux. That figure provides a reference when considering additional direct sources of energy flux being deposited into the atmosphere.

Fitting a function of the form

$$R(T) = a - \frac{b}{T - 250} \quad (13)$$

to the data we used to draw the natural transmittance curve of Figure 7, deduced as said from the results presented in Table 2, with the tem-

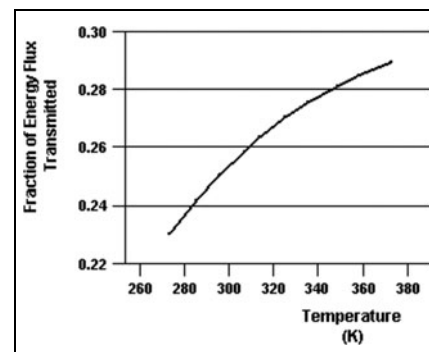


Figure 7. The ratio of the energy flux generated by a blackbody at a given temperature which is transmitted through the atmosphere to the energy flux of the blackbody provides a meaningful measure of the nature of the transmittance in the range of infrared wavelengths from 0 to $600\ \mu$ for natural objects. We can refer to this, in contrast to that depicted in Figure 5 and Figure 6, as the natural transmittance of the atmosphere.

perature measured in Kelvin, we find a best fit for

$$a = 0.298 \pm 0.002$$

$$b = 1.917 \pm 0.005\ K,$$

with a standard error of estimate of 0.0048 based on two degrees of freedom resulting from the use of the two parameters. This indicates an excellent fit. That the value of a resulting from the fit is similar to the numbers provided in the ordinate scale of Figure 7 justify choosing the form of equation (13).